III. "A Dynamical Theory of the Electric and Luminiferous Medium." By JOSEPH LARMOR, F.R.S., Fellow of St. John's College, Cambridge. Received November 15, 1893.

(Abstract.)

Ever since the causes of natural phenomena began to attract attention, the interaction of the different classes of physical agencies has been taken to suggest that they are all manifestations in different ways of the energy of some fundamental medium; and the efforts of the more sanguine class of naturalists have always been in some measure directed towards the discovery of the properties of this medium. It is only at the end of the last century that the somewhat vague principle of the economy of action or effort in physical actions —which, like all other general principles in the scientific explanation of Nature, is ultimately traceable to a kind of metaphysical origin— has culminated in the hands of Lagrange in his magnificent mathematical generalisation of the dynamical laws of material systems. Before the date of this concise and all-embracing formulation of the laws of dynamics there was not available any engine of sufficient power and generality to allow of a thorough and exact exploration of the properties of an ultimate medium, of which the mechanism and mode of action are almost wholly concealed from view. The precise force of Lagrange's method, in its physical application, consists in its allowing us to ignore or leave out of account altogether the details of the mechanism, whatever it is, that is in operation in the phenomena under discussion; it makes everything depend on a single analytical function representing the distribution of energy in the medium in terms of suitable co-ordinates of position and of their velocities; from the location of this energy, its subsequent play and the dynamical phenomena involved in it are all deducible by straightforward mathematical analysis.

The problem of the correlation of the physical forces is thus divisible into two parts, (i) the determination of the analytical function which represents the distribution of energy in the primordial medium which is assumed to be the ultimate seat of all phenomena, and (ii) the discussion of what properties may be most conveniently and simply assigned to this medium, in order to describe the play of energy in it most vividly, in terms of the stock of notions which we have derived from the observation of that part of the interaction of natural forces which presents itself directly to our senses, and is formulated under the name of natural law. It may be held that the first part really involves in itself the solution of the whole problem; that the second part is rather of the nature of illustration and ex-

planation, by comparison of the intangible primordial medium with other dynamical systems of which we can directly observe the phenomena.

The chief representative of exact physical speculation of the second of these types has been Lord Kelvin. In the older attempts of this kind the dynamical basis of theories of the constitution of the æther consisted usually in a play of forces, acting at a distance, between ultimate elements of molecules of the medium; from this we must, however, except the speculations of Greek philosophy and the continuous vortical theories of the school of Descartes, which were of necessity purely descriptive and imaginative, not built in a connected manner on any rational foundation. It has been in particular the aim of Lord Kelvin to deduce material phenomena from the play of inertia involved in the motion of a structureless primordial fluid; if this were achieved it would reduce the duality, rather the many-sidedness, of physical phenomena to a simple unity of scheme; it would be the ultimate conceivable simplification. The celebrated vortex theory of matter makes the indestructible material atoms consist in vortex rings in a primordial fluid medium, structureless, homogeneous, and frictionless, and makes the forces between the atoms which form the groundwork of less fundamental theories consist in the actions excited by these vortices on one another through the inertia of the fluid which is their basis—actions which are instantaneously transmitted if the fluid is supposed to be absolutely incompressible.

In case this foundation proves insufficient, there is another idea of Lord Kelvin's by which it may be supplemented. The characteristic properties of radiation, which forms so prominent an element in actual phenomena, can be explained by the existence of an elastic medium for its transmission at a finite, though very great, speed; such a medium renders an excellent account of all its relations, if we assume it to possess inertia and to be endowed with some elastic quality of resistance to disturbance roughly analogous to what we can observe and study in ordinary elastic solids of the relatively incompressible kind, such as india-rubber and jellies. Lord Kelvin has been the promoter and developer of a view by which the elastic forces between parts of such a medium may be to some extent got rid of as ultimate elements, and be explained by the inertia of a spinning motion of a dynamically permanent kind, which is distributed throughout its volume. If we imagine very minute rapidly-spinning fly-wheels or gyrostats spread through the medium, they will retain their motion for ever, in the absence of friction on their axles, and they will thus form a concrete dynamical illustration of a type of elasticity which arises solely from inertia; and this illustration will be of great use in realising some of the peculiarities of a related

type, which I believe can be thoroughly established as the actual type of elasticity transmitting all radiations, whether luminous and thermal or electrical—for they are all one and the same—through the ultimate medium of fluid character of which the vortices constitute matter.

It has always been the great puzzle of theories of radiation how the medium which conveys it by transverse vibrations, such as we know directly only in media of the elastic-solid type, could yet be so yielding as to admit of the motion of the heavenly bodies through it absolutely without resistance. According to the view of the constitution of the æther which is developed in this paper, not only are these different properties absolutely consistent with each other, but it is, in fact, their absolute and rigorous coexistence which endows the medium with the qualities necessary for the explanation of a further very wide class of phenomena. The remark which is the key to this matter has been already thrown out by Lord Kelvin, in connexion with Sir George Stokes's suggested explanation of the astronomical aberration of light. The motion of the ultimate homogeneous frictionless fluid medium, conditioned by the motion of the vortices existing in it, is, outside these vortices, of an absolutely irrotational character. Now, suppose the medium is endowed with elasticity of a purely rotational type, so that its elastic quality can be called into play only by absolute rotational *displacement* of the elements of the medium; just as motion of translation of a spinning gyrostat calls into play no reaction, while any alteration of the absolute position of its axis in space is resisted by an opposing couple. As regards the motion of the medium involved in the movements of its vortices, this rotational elasticity remains completely latent, as if it did not exist; and we can at once set down the whole theory of the vortical hydrodynamical constitution of matter as a part of the manifestations of an ultimate medium of this kind.

We have now to indicate some of the consequences of the assumed constitution of the æther as regards the phenomena of radiation, which depend on this elasticity: to do this it will be convenient to make a fresh start, dealing more particularly with the first part of the general question.

The true nature of the phenomena of light had been brought to view at the beginning of the present century by the intuition of Thomas Young; and the secret of the exact quantitative mathematical laws which govern the behaviour of light in all the various circumstances attending its propagation, reflexion, and refraction had been fathomed in a marvellous manner by the genius of Fresnel. The nature of the mathematical reasoning by which Fresnel was led to his results has for the most part never been understood; and, as presented by him in his writings, it certainly seems devoid of

dynamical coherence and formal logical validity. Yet, the more the phenomena of light were afterwards experimentally examined, the stronger was the confirmation of the whole scheme of formulæ at which he had arrived.

The explanation of the laws of physical optics advanced by Fresnel, and verified by comparison with the phenomena, which was possible in several very exact ways, chiefly by himself and Brewster, was, about the year 1835, engaging the attention of several of the chief mathematicians of that time—Augustin Cauchy in France, Franz Neumann in Germany, George Green in England, and James MacCullagh in Ireland. The prevalent mode of attacking the problem was through the analogy with the propagation of elastic waves in solid bodies; and the comparison of Fresnel's laws of propagation in crystalline media with the results of the mathematical theory of the elasticity of crystalline bodies gave abundance of crucial tests for the verification, modification, or disproof of the principles assumed in these investigations. The treatment of Cauchy is earliest in date, but somewhat empirical and unsatisfactory in its logical aspects in the light of subsequent more precise knowledge of the conditions of the problem of the elasticity of solids. The treatment of Neumann is also a sound and original piece of investigation, if we except the limited view of the elasticity of solids, that of Navier and Poisson, on which he based it. The treatment by Green had the great distinction of incidentally laying, with all the generality and simplicity which we expect in an ultimate theory, the foundations on which every theory of elastic action in ordinary material bodies must in future be constructed; it proceeded, in fact, on the basis of one of those great generalisations, of which the aggregate constitutes the all-embracing modern doctrine of energy. These three authors all treated the question of reflexion and refraction of waves. Cauchy could not make much of Fresnel's formulæ in any logical manner. Neumann had the merit of seeing clearly that the thing was impossible on his elastic solid theory; so he dropped it altogether, assumed a sufficient number of principles which might be taken, with fair probability, in accordance with general reasoning, to be satisfied in the reflexion and refraction of light rays, viz., complete continuity of the media and continuity of energy in crossing the boundary at which the reflexion and refraction take place, and had the satisfaction of evolving a solution which agreed with Fresnel's laws, and easily extended them to the much more complicated circumstances of crystalline media. But to obtain this solution he assumed, from what he found necessary to make his very imperfect theory of propagation in crystals agree with Fresnel's laws, that the density of the luminiferous medium is the same in all bodies, and that the displacement of plane-polarised light is in the plane of polarisation. It may

be shown, as is now indeed to be expected, that this is a totally wrong foundation to work upon, that Neumann's general principles for the solution of the problem of reflexion are inconsistent with his elastic theory. If he had adopted a converse procedure, and worked out the problem of the reflexion of a ray on his general principles, and then deduced, by comparison with Fresnel's formulæ, the law of density of the luminiferous medium and the direction of the vibration in plane-polarised light, he would have been entitled to the credit of a joint discoverer in the domain of the dynamics of reflexion. But, for the reasons here indicated, the credit of that discovery must, I think, be assigned to MacCullagh.

The achievements by which the memory of MacCullagh is now to a great extent preserved are his very elegant investigations in the domain of pure Euclidian geometry. He may be claimed to be an instance of the numerous cases from Archimedes down through Descartes, Newton, and, we may add, Thomas Young, in which keen geometrical insight has formed a key for unlocking the formal laws of physical actions. He was first attracted to Fresnel's laws of optics by the very simple and elegant geometrical relations to which they lead. At a later period he proposed to himself the problem to hit off the extension of Fresnel's laws of reflexion which would apply to crystalline media, in the light of the crucial conditions afforded by the delicate experiments of Brewster and, at a later stage, Seebeck, to which such a theory must conform. He had thus to cast about for geometrical principles on which Fresnel's laws might be founded, such as would admit of easy extension to the more general problem. He early came upon the principle of continuity of the media, which he put in the geometrical form that the resultant of the displacements in the refracted waves is equal to the resultant of the displacements in the incident and reflected waves. As regards the other necessary condition, he was not at first successful. The density of the medium he took to be the same in all bodies, because he could not imagine it to be æolotropic, or different in different directions, in crystalline media. He assumed the vibrations to be in the plane of polarisation, from considerations of geometrical symmetry and necessity, confirmed in the earlier stage by one of the theories of Cauchy. The other condition above referred to he took to be equality of certain pressures in the media, as imagined by Cauchy; and by this means he arrived at a satisfactory explanation of Brewster's observations on the polarising angle in reflexion from crystals. But Seebeck pointed out that this solution would not account for the values of the deviation of the plane of polarisation from the plane of reflexion, by means of which he had himself tested it. Owing to this criticism MacCullagh was finally led to abolish Cauchy's notion of pressure, and assume simply the continuity of energy in its place.

This principle of energy, which gives a quadratic equation between the displacements at the interface, he succeeded to his satisfaction, as regards the confirmation of his views, in replacing by linear relations. And then he gave his two magnificent geometrical theorems—that of *transversals* and that of the *polar plane*, which contain each in a sentence the complete specification of the laws of reflexion for the most general case of a transparent medium, and which form the culmination of the geometrical relations by which he was guided throughout this whole process of synthetical discovery. His laws of reflexion are the same as Neumann's; of them, as formal laws, these two authors must be regarded as the independent discoverers—Neumann by a happy assumption suggested by reasoning at bottom illogical in the light of subsequent knowledge, MacCullagh by a resolute attack on the observed facts with a view to reducing them to simple formulæ.

But the greatest achievement of MacCullagh is that contained in his memoir of 1839, two years after, entitled an "Essay towards a Dynamical Theory of Crystalline Reflexion and Refraction." He is in quest of a dynamical foundation for the whole scheme of optical laws, which had been notably extended and confirmed by himself already. He recognises, I think for the first time in a capital physical problem, that what is required is the discovery of the potential-energy function of Lagrange on which the action of the medium depends, and that the explanation of the form of that function is another question which can be treated separately. His memoir is subsequent to, but apparently quite independent of, that of Green, in which Green restricted the medium to a constitution like an elastic solid, laid down the general laws of such constitution for the first time, and made a magnificent failure of his attempt to explain optical phenomena on that basis. If this thing was to be done, the power, simplicity, and logical rigour of Green's analysis might have been expected to do it; and nothing further has come of the matter until the recent new departure of Lord Kelvin in his speculation as to a labile elastic-solid æther. To return to MacCullagh, he is easily able to hit off a simple form of the potential-energy function, which—on the basis of Lagrange's general dynamics, or more compactly on the basis of the law of Least Action—absolutely sweeps the whole field of optical theory so far as all phenomena are concerned in which absorption of the light does not play a prominent part. He is confident, as any one who follows him in detail must be, that he is on the right track. He tries hard to obtain a dynamical basis for his energy-function, that is, to imagine some material medium that shall serve as a model for it and illustrate its possibility and its mode of action; he records his failure in this respect, but at the same time he protests against the limited view which would tie

down the unknown and in several ways mysterious and paradoxical properties of the luminiferous medium to be the same as those of an ordinary elastic solid.

The form of MacCullagh's energy-function was derived by him very easily from the consideration of the fact that it is required of it that it shall produce, in crystalline media, plane-polarised waves propagated by displacements in the plane of the wave front. Though he seems to put his reasoning as demonstrative on this point, it has been pointed out by Sir George Stokes, and is indeed obvious at once from Green's results, that other forms of the energy-function besides MacCullagh's would satisfy this condition. But the important point as regards MacCullagh's function is that it makes the energy in the medium depend solely on the absolute rotational displacements of its elements from their equilibrium orientations, not at all on its distortion or compression, which are the quantities on which the elasticity of a solid would depend according to Green.

Starting from this conception of rotational elasticity, it can be shown that, if we neglect for the moment optical dispersion, every crystalline optical medium has three principal elastic axes, and its wave-surface is precisely that of Fresnel, while the laws of reflexion and refraction agree precisely with experiment. Further, it follows from the observed fact of transparency in combination with dispersion, that the dispersion of a wave of permanent type is properly accounted for by the addition to the equations, therefore to the energy-function, of subsidiary terms involving spacial differentiations of higher order. To preserve the medium hydrodynamically a perfect fluid, these terms also must satisfy the condition that the elasticity of the medium is thoroughly independent of compression and distortion of its elements, and wholly dependent on absolute rotation. It can be shown, I believe, that this restriction limits the terms to two kinds, one of which retains Fresnel's wave surface unaltered, while the other modifies it in a definite manner stated without proof by MacCullagh [; but the first terms depend on an interaction between the dispersive property and the wave motion itself, while the second terms involve the square of the dispersive quality. It seems clear that the second type involves only phenomena of a higher order of small quantities than we are here considering—*December* 7, 1893]; thus an account of dispersion remains which retains Fresnel's wave surface unaltered for each homogeneous constituent of the light, while it includes the dispersion of the axes of optical symmetry in crystals as regards both their magnitudes and directions—results quite unapproached by any other theory ever entertained.

In this analysis of dispersions, all terms have been omitted which possess a unilateral character, such as would be indicated in actuality by rotatory polarisation and other such phenomena. The laws of

crystalline material structures seem to prohibit the occurrence of such asymmetry as these terms would indicate, except to the very small extent evidenced by the hemihedral faces of quartz crystals. The influence on the optical medium of this asymmetric arrangement of the molecules must be very much smaller still, for the rotatory terms are in all media exceedingly minute compared with the ordinary dispersional terms. The form of these rotatory terms in the energy-function is at once definitely assigned by our condition of perfect fluidity of the medium, both for crystals and for rotational liquids such as turpentine, and this form is the one usually accepted, on MacCullagh's suggestion, as yielding a correct account of the phenomena.

When dispersional terms are included in the energy function, our continuous analysis is not any longer applicable to the problem of reflexion; the conditions at the interface are altogether too numerous to be satisfied by the available variables. There is in fact discontinuity at the interface in the discrete molecular structure, such as could not be representable by a continuous analysis. But if we proceed by the method of rays, and assume that there is a play of surface forces which do not absorb any energy, while they adjust the dispersional part of the stress, it appears that reflexion is independent of dispersion.

The treatment of the problem of reflexion by Fresnel involves a different direction of vibration of the light, and different surface conditions, from MacCullagh's. It is of interest to remark that this theory may be stated in a dynamically rigorous form, provided the medium to which it refers possesses the properties of the labile elastic-solid æther of Lord Kelvin; and Fresnel's own account of his analysis of the problem becomes more intelligible from such a standpoint.

The mention of the phenomena of magnetic rotational quality will introduce us to the next division of the subject, that of the inclusion of electric and magnetic phenomena in the domain of the activity of this primordial medium.

The problem of the æther has been first determinedly attacked from the side of electrical phenomena by Clerk Maxwell in quite recent times; his great memoir on a 'Dynamical Theory of the Electromagnetic Field' is of date 1864. It is in fact only comparatively recently that the observation of Oersted, and the discoveries and deductions of Ampère, Faraday, and Thomson had accumulated sufficient material to allow the question to be profitably attacked from this side. Even as it is, our notions of what constitute electric and magnetic phenomena are of the vaguest as compared with our ideas of what constitutes radiation, so that Maxwell's views involve difficulties, not to say contradictions, and in places present obstacles

which are to be surmounted, not by logical argument or any clear representation, but by the physical intuition of a mind saturated with this aspect of the phenomena. Many of these obstacles may, I think, be removed by beginning at the other end, by explaining electric actions on the basis of a mechanical theory of radiation, instead of radiation on the basis of electric actions. The strong point of Maxwell's theory is the electromotive part, which gives an account of electric radiation and of the phenomena of electromagnetic induction in fixed conductors; and this is in keeping with the remark just made. The nature of electric displacement, of electric and magnetic forces on matter, of what Maxwell calls the electrostatic and the magnetic stress in the medium, of electrochemical phenomena, are all left obscure.

We shall plunge into the subject at once from the optical side, if we assume that dielectric polarisation consists in a strain in the æther, of the rotational character contemplated above. The conditions of internal equilibrium of a medium so strained are easily worked out from MacCullagh's expression for W, its potential energy. If the vector (f, g, h) denote the curl or vorticity of the actual linear displacement of the medium, or *twice* the absolute rotation of the portion of the medium at the point considered, and the medium is supposed of crystalline quality and referred to its principal axes, so that

$$W = \tfrac{1}{2}\int (a^2f^2+b^2g^2+c^2h^2)\,d\tau,$$

where $d\tau$ is an element of volume, it follows easily that for internal equilibrium we must have

$$a^2f\,dx+b^2g\,dy+c^2h\,dz = -dV,$$

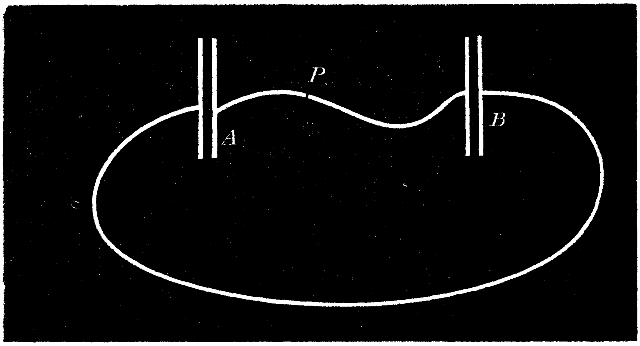
a complete differential, and that over any boundary enclosing a region devoid of elasticity the value of V must be constant. Such a boundary is the surface of a conductor; V is the electric potential in the field due to charges on the conductors; (f, g, h) is the electric displacement in the field, circuital by its very nature as a rotation, and (a^2f, b^2g, c^2h) is the electric force derived from the electric potential V. The charge on a conductor is the integral of (f, g, h) over any surface enclosing it, and cannot be altered except by opening up a channel devoid of elasticity, in the medium, between this conductor and some other one; in other words, electric discharge can take place only by rupture of the elastic quality of the æthereal medium.

[At the interface between two dielectric media, taken to be crystalline as above, the condition comes out to be that the tangential electric force is continuous. When the circumstances are those of equilibrium, and therefore an electric potential may be introduced, this condition allows discontinuity in the value of the potential in crossing

the interface, but demands that the amount of this discontinuity shall be the same all along the interface; these are precisely the circumstances of the observed phenomena of voltaic potential differences. The component, normal to the interface, of the electric displacement is of course always continuous, from the nature of that vector as a flux.

It may present itself as a difficulty in this theory that, as the electric displacement is the rotational displacement of the medium, its surface integral over any sheet should be equal to the line integral of the linear displacement of the medium round the edge of the sheet; therefore that for a closed sheet surrounding a conductor this integral should be null, which would involve the consequence that the electric charge on a conductor cannot be different from null. This line of argument, however, implies that the linear displacement is a perfectly continuous one, which is concomitant with and required by the electric displacement. The legitimate inference is that the electric displacement in the medium which corresponds to an actual charge cannot be set up without some kind of discontinuity or slip in the linear displacement of the medium; in other words, that a conductor cannot receive an electric charge without rupture of the surrounding medium; nor can it lose a charge once received without a similar rupture. The part of the linear displacement that remains, after this slip or rupture has been deducted from it, is of elastic origin, and must satisfy the equations of equilibrium of the medium. —*December* 7, 1893.]

We can produce in imagination a steady electric current, without introducing the complication of galvanic batteries, in the following manner, and thus examine in detail all that is involved, on the present theory, in the notion of a current. Suppose we have two charged condensers, with one pair of coatings connected by a narrow conducting channel, and the other pair connected by another such channel, as in the annexed diagram, where the dark regions are dielectric and the white regions conducting. If we steadily move towards each other the two plates of the condenser A, a current will flow round the circuit, in the form of a conduction current in the conductors and a displacement current across the dielectric plates of the condensers. Let us suppose the thicknesses of these dielectric plates to be excessively small, so as to minimise the importance of the displacement part of the current. There is then practically no electric force, and therefore no electric displacement, in the surrounding dielectric field, except between the plates of the condensers and close to the conducting wires. Consider a closed surface passing between the faces of the condenser A, and intersecting the wire at a place P. A movement of the faces of this condenser alters the electric force between them, and therefore alters the electric displacement across the portion of



this closed surface which lies in that part of the field; as we have seen there is practically no displacement, anywhere else in the field except at the conducting wire; therefore to preserve the law of the circuital character of displacement throughout the whole space, we must suppose that this alteration is compensated by a very intense change of displacement at the conducting wire. So long as the movement of the plates continues, as long does this flow of displacement along the wire go on; it constitutes the electric current in the wire. Now, in calculating the magnetic force in the field, which is the velocity of the æthereal medium, from the change of electric displacement, we must include in our integration the effect of this sheet of electric displacement flowing along the surface of the perfectly conducting wires, for exactly the same reason as in the correlative problem in hydrodynamics, of calculating the velocity of the fluid from the distribution of vorticity in it, Helmholtz had to consider a vortex sheet as existing over each surface across which the motion is discontinuous.

The next stage in this mode of elucidation of electrical phenomena is to suppose, once the current is started in our non-dissipative circuit, that both the condensers are instantaneously removed, and replaced by continuity of the wire. We are now left with a current circulating round a complete perfectly conducting channel, which in the absence of viscous forces will flow round permanently. The expression for the kinetic energy in the field is easily transformed from a volume integral of the magnetic force, which is represented by the velocity of the medium $\frac{d}{dt}(\xi, \eta, \zeta)$, to an integral involving the current $\frac{d}{dt}(f, g, h)$, which is in the present case a line integral round the electric circuit. The result is Franz Neumann's celebrated formula for the electromagnetic energy of a linear electric current,

$$\mathrm{T} = \tfrac{1}{2}\iota^2 \iint r^{-1} \cos \epsilon \, ds \, ds;$$

or we may take the case of several linear circuits in the field, and obtain the formula

$$T = \tfrac{1}{2}\Sigma\iota^2 \iint r^{-1} \cos\epsilon\, ds\, ds + \Sigma\iota_1\iota_2 \iint r^{-1} \cos\epsilon\, ds_1\, ds_2,$$

which is sufficiently general to cover the whole ground of electro-dynamics.

Our result is in fact that a linear current is a vortex ring in the fluid æther, that electric current is represented by vorticity in the medium, and magnetic force by the velocity of the medium. The current being carried by a perfect conductor, the corresponding vortex is (as yet) without a core, *i.e.*, it circulates round a vacuous space. [The strength of a vortex ring is, however, permanently constant; therefore, owing to the mechanical connexions and continuity of the medium, a current flowing round a complete perfectly conducting circuit would be unaffected in value by electric forces induced in the circuit, and would remain constant throughout all time. Ordinary electric currents must therefore be held to flow in incomplete conducting circuits, and to be completed either by convection across an electrolyte or by electric displacement or discharge across the intervals between the molecules, after the manner of the illustration given above.—*December* 7, 1893.]

Now we are here driven upon Ampère's theory of magnetism. Each vortex-atom in the medium is a permanent non-dissipative electric current of this kind, and we are in a position to appreciate the importance which Faraday attached to his discovery that all matter is magnetic. Indeed, on consideration, no other view than this seems tenable; for we can hardly suppose that so prominent a quality of iron as its magnetism completely disappears above the temperature of recalescence, to reappear again immediately the iron comes below that temperature; much the more reasonable view is that the molecular rearrangement that takes place at that temperature simply masks the permanent magnetic quality. In all substances other than the magnetic metals, the vortex atoms pair into molecules and molecular aggregates in such way as to a large extent cancel each other's magnetic fields; why in iron at ordinary temperatures the molecular aggregates form so striking an exception to the general rule is for some reason peculiar to the substance, which, considering the complex character of molecular aggregation in solids, need not excite surprise.

We have now to consider the cause of the pairing together of atoms into molecules. It cannot be on account of the magnetic, *i.e.*, hydrodynamical, forces they exert on one another, for two electric currents would then come together so as always to reinforce each other's magnetic action, and all substances would be strongly magnetic. The ionic electric charge, which the phenomena of electrolysis show to

exist on the atom, supplies the attracting agency. Furthermore, the law of attraction between these charges is that of the inverse square of the distance, and between the atomic currents is that of the inverse cube; so that, as in the equilibrium state of the molecule these forces are of the same order of intensity and counteract each other, the first force must have much the longer range, and the energy of chemical combination must therefore be very largely electrostatic, due to the attraction of the ions, as von Helmholtz has clearly made out from the phenomena of electrolysis and electrolytic polarisation.

But in this discussion of the phenomena of chemical combination of atoms we have been anticipating somewhat. All our conclusions, hitherto, relate to the æther, and are therefore about electromotive forces. We have not yet made out why two sets of molecular aggregates, such as constitute material bodies, should attract or repel each other when they are charged, or when electric currents circulate in them; we have, in other words, now to explain the electrostatic and electrodynamic forces which act between material systems.

Consider two charged conductors in the field; for simplicity, let their conducting quality be perfect as regards the very slow displacements of them which are contemplated in this argument. The charges will then always reside on their surfaces, and the state of the electric field will, at each instant, be one of equilibrium. The magnitude of the charge on either conductor cannot alter by any action short of a rupture in the elastic quality in the æther; but the result of movement of the conductors is to cause a re-arrangement of the charge on each conductor, and of the electric displacement (f, g, h) in the field. Now the electric energy W of the system is altered by the movement of the conductors, and no viscous forces are in action; therefore the energy that is lost to the electric field must have been somehow spent in doing mechanical work on the conductors; the loss of potential energy of the electric field reappears as a gain of potential energy of the conductors. We have to consider how this transformation is brought about. The movement of the conductors involves, while it lasts, a very intense ideal flow of electric displacement along their surfaces, and also a real change of displacement of ordinary intensity throughout the dielectric. The intense surface flow is in close proximity with the electric flows round the vortex atoms which lie at the surface; their interaction produces a very intense elastic disturbance in the medium, close at the surface of the conductor, which is distributed by radiation through the dielectric as fast as it is produced; the elastic condition of the dielectric, on account of its extreme rapidity of propagation of disturbances compared with its finite extent, being always extremely nearly one of equilibrium. It is, I believe, the reaction on the conductor of these wavelets which are continually shooting out from its surface, carry-

ing energy into the dielectric, that constitutes the mechanical forcive acting on it. But we can go further than this; the locality of this transformation of energy, so far at any rate as regards the material forcive, is the surface of the conductor; and the gain of mechanical energy by the conductor is therefore correctly located as an absorption of energy at its surface; therefore the forcive acting on the conductor is correctly determined as a surface traction, and not a bodily forcive throughout its volume. One mode of representing the distribution of this surface traction, which, as we know, gives the correct amount of work for every possible kind of virtual displacement of the surface, is to consider it in the ordinary electrostatic manner as a normal traction due to the action of the electric force on the electric density at the surface; we conclude that this distribution of traction is the actual one. To recapitulate: if the dielectric did not transmit disturbance so rapidly, the result of the commotion at the surface produced by the motion of the conductor would be to continually start wavelets which would travel into the dielectric, carrying energy with them. But the very great velocity of propagation effectually prevents the elastic quality of the medium from getting hold; no sensible wave is produced and no flow of energy occurs into the dielectric. The distribution of pressure in the medium which would be the accompaniment of the wave motion still persists, though it now does no work in the dielectric; it is this pressure of the medium against the conductor that is the cause of the mechanical forcive.

The matter is precisely illustrated by the fundamental *aperçu* of Sir George Stokes with regard to the communication of vibrations to the air or other gas. The rapid vibrations of a tuning-fork are communicated as sound waves, but much less completely to a mobile medium like hydrogen than to air. The slow vibrations of a pendulum are not communicated as sound waves at all; the vibrating body cannot get a hold on the elasticity of the medium, which retreats before it, preserving the equilibrium condition appropriate to the configuration at the instant; there is a pressure between them, but this is instantaneously equalised throughout the medium as it is produced, without leading to any flow of vibrational energy.

Now let us formally consider the dynamical system consisting of the dielectric media alone, and having a boundary just inside the surface of each conductor; and let us contemplate motions of the conductors so slow that the medium is always indefinitely near the state of internal equilibrium or steady motion, that is conditioned at each instant by the position and motion of the boundaries. The kinetic energy T of the medium is the electrodynamic energy of the currents, as given by Neumann's formula; and the potential energy W is the energy of the electrostatic distribution corresponding to the conformation at the instant; in addition to these energies we shall have to

take into account surface tractions exerted by the enclosed conductors on the medium, at its boundaries aforesaid. The form of the general dynamical variational equation that is suitable to this problem is

$$\delta\int(\mathrm{T}-\mathrm{W})\,dt+\int dt\int\delta w\,d\mathrm{S}=0,$$

where $\delta w\,d\mathrm{S}$ represents the work done by the tractions acting on the element $d\mathrm{S}$ of the boundary, in the virtual displacement contemplated. If there are electromotive sources in certain circuits of the system, which are considered to introduce energy into it from outside itself, the right-hand side of this equation must also contain an expression for the work done by them in the virtual displacement contemplated of the electric coordinates. Now this variational equation can be expressed in terms of any generalised coordinates whatever, that are sufficient to determine the configuration in accordance with what we know of its properties. If we suppose such a mode of expression adopted, then, on conducting the variation in the usual manner and equating the coefficients of each arbitrary variation of a coordinate, we obtain the formulæ

$$\Phi=\frac{d}{dt}\frac{d\mathrm{T}}{d\dot{\phi}}-\frac{d\mathrm{T}}{d\phi}+\frac{d\mathrm{W}}{d\phi},$$

$$\mathrm{E}=\frac{d}{dt}\frac{d\mathrm{T}}{d\dot{e}}.$$

In these equations Φ is a component of the mechanical forcive exerted on our dielectric system by the conductors, as specified by the rule that the work done by it in a displacement of the system represented by $\delta\phi$, a variation of a single coordinate, is $\Phi\delta\phi$: the corresponding component of the forcive exerted by the dielectric system on the conductor is of course $-\Phi$. Also E is the electromotive force which acts from outside the system in a circuit in which the electric displacement is e, so that the current in it is $\dot{e}$; the electromotive force induced in this circuit by the dielectric system is $-\mathrm{E}$.

These equations involve the whole of the phenomena of ordinary electrodynamic actions, whether ponderomotive or electromotive, whether the conductors are fixed or in motion through the medium: in fact, in the latter respect no distinction appears between the cases. They will be completed presently by taking account of the dissipation which occurs in ordinary conductors.

These equations also involve the expressions for the electrostatic ponderomotive forces, the genesis of which we have already attempted to trace in detail. The generalised component, corresponding to the co-ordinate ϕ, of the electrostatic traction of the conductors on the dielectric system, is $d\mathrm{W}/d\phi$; therefore the component of the traction,

somehow produced, of the dielectric system on the conductors is $-dW/d\phi$.

The stress *in the æther* between two electrified bodies consists of a tangential traction on each element of area, equal in magnitude to the tangential component of the electric force at that place, and at right angles to its direction. The stress in the *material* of the dielectric is such as is produced in the ordinary manner by the surface tractions exerted on the material by the conductors that are imbedded in it. The stress in the dielectric of Faraday and Maxwell has no real existence; it is in fact such a stress as would be felt by the surface of a conductor used to explore the field, when the conductor is so formed and placed as not to disturb the electric force in the dielectric. The magnetic stress of Maxwell is simply a mathematical mode of expression of the kinetic reaction of the medium.

The transfer of a charged body across the field with velocity not large compared with the velocity of electric propagation carries with it the whole system of electric displacement belonging to the body, and therefore produces while it lasts a system of displacement currents in the medium, of which the circuits are completed by the actual flow of charge along the lines of motion of the different charged elements of the body.

The phenomena of the electrostatic polarisation of dielectrics were at one time provisionally represented by Faraday as due to the orientation of electric polar elements of the medium by the electric force, just as magnetisation is actually due to the orientation of the magnetic polar elements by the magnetic force of the field; and this theory was developed at length by Mossotti. At a later period Maxwell himself ('Dynamical Theory,' § 11) compared the electric displacement in a dielectric medium to an actual displacement of the electric charge on conducting molecules imbedded in it—a conception mathematically equivalent to the above. In a previous paper* I have explained by simple reasoning that this view is inconsistent with the circuital character of the electric current, a conclusion in agreement with that of von Helmholtz, who adopted this idea in his generalised theory of electrodynamics. It is therefore necessary to obtain a complete view of this matter from our present standpoint. The polarised molecule, with its positive and negative ions, is as we have seen a reality; but if the current is to remain circuital, the action of the electric force of the field must not affect the actions between the constituent vortices which are the cause of their orientation, nor the distribution of the electric charges on the atoms, so much as to produce any sensible electric displacement of this kind. These restrictions might be secured by taking the two poles of the molecule

* "On the Theory of Electrodynamics," 'Roy. Soc. Proc.,' vol. 49, 1891, p. 522.

sufficiently close together, and by taking the dimensions of the conducting atom sufficiently small.

As regards the second of these hypotheses, it is to be observed that the moment of electric induction in a conducting atom depends only on its size, and not on the intensity of its free electrification; for the case of conducting spherules the electric moment produced by the action of an electric force F is $3F/4\pi$ multiplied by the total volume of the atoms, and this would give a dielectric inductive coefficient equal to three times the ratio of the aggregate volume of the atoms to the whole volume of the region, a result which is, in any case, far too small to represent the facts, and may easily be so small as to be quite negligible, so as to leave the current practically circuital.

But if we add on to this the first assumption, no room will be left for the explanation of the pyro-electricity and piezo-electricity of crystalline media, by changes of orientation of polar molecules due to changes of temperature or to applied pressure. If this very rational explanation is to be retained, we are driven to assume that the electric force of the field does not sensibly alter the orientation of a molecule, which would then be wholly controlled by the internal electrical, chemical, and cohesive forces of the medium. The state of matters thus required is, in fact, precisely realised by a symmetrical arrangement of positive and negative atoms in the molecule, such as the hexagonal molecule recently imagined by Lord Kelvin,* and earlier by J. and P. Curie, to account for the piezo-electric quality of quartz; the symmetry of the electric charges makes null the aggregate electric moment and therefore the turning couple in an electric field, while a differential polarity can still be developed under strain of the crystal.

According to the present theory of electrification, a discharge of electricity from one conductor to another can only occur by the breaking down of the elasticity of the dielectric æther along some channel connecting them; and a similar rupture is required to explain the transfer of an atomic charge to the electrode in the phenomenon of electrolysis. We can conceive the polarisation increasing by the accumulation of dissociated ions at the two electrodes of a voltameter, until the stress in the portion of the medium between the ions and the conducting plate breaks down, and a path of discharge is opened from some ion to the plate. While this ion retained its charge, it repelled its neighbours; but now electric attraction will ensue, and the one that gets into chemical contact with it first will be paired with it by the chemical forces; while if the conducting path to the electrode remains open until this union is complete, the ion will receive an opposite atomic charge from the electrode, which very conceivably may have to be also of equal amount, in order to equalise

* Lord Kelvin, 'Phil. Mag.,' October, 1893.

the potentials of the molecule and the plate. This is on the hypothesis that the distance between the two ions of a molecule is very small compared with the distance between two neighbouring molecules. A view of this kind, if thoroughly established, would lead to the ultimate averaging of atomic charges of all atoms that have been in combination with each other, even if those charges had been originally of different magnitudes. [The assignment of free electric charges to vortex atoms tends markedly in the direction of instability; though instability under certain circumstances is essential to electric discharge, yet it must not be allowed to become dominant.—*December* 7, 1893.]

The presence of vortex atoms, forming faults so to speak in the æther, will clearly diminish its effective rotational elasticity; and thus it is to be expected that the specific inductive capacities of material dielectrics should be greater than the inductive capacity of a vacuum. The readiness with which electrolytic media break down under electric stress may be connected with the extremely high values of their inductive capacities, indicating very great yielding to even a small electric force.

The radiation of a body into the surrounding medium is wholly electrical, and is due to the electric vibrations of the atomic charges; some of these types of vibration may correspond to the single atom by itself, while others will be considerably affected by the presence of the neighbouring atoms of the molecule. The most striking fact to be explained is the total independence of temperature that is exhibited by the periodic times corresponding to the various spectral lines. The extreme smallness of an atom implies correspondingly intense electrification, and therefore independence of the external field. If it is assumed that the dimensions and configuration of the atom are determined by the very intense actions between it and its partners in the molecule, and are not sensibly affected by the comparatively feeble influence of the velocity of translation of the molecule through the medium, this fact will be accounted for; irregularities can then only occur during an encounter with another molecule.

In the hydrodynamics of ordinary liquids, when the energy of an isolated vortex ring is increased, the ring expands in radius, and therefore moves onward more slowly. But in the case of an isolated charged atom, an expansion in radius diminishes the potential energy of the electric charge. These two agencies counteract each other; if the latter one is the greater, increase in energy will involve increase in velocity, as would be required in the ordinary form of the kinetic theory of gases. But the more natural supposition is, perhaps, to consider a molecule as composed of atoms paired so that the velocity of translation does not depend intrinsically on the amount

of energy associated with the molecule, but is determined by the circumstances of the encounters with other molecules. The distribution of energy between the various vibration-types of the molecule, according to the law of Maxwell and Boltzmann, will not affect its configuration, while there is also perfect independence between the hydrodynamical motion of the medium due to the molecule and the radiation produced by it.

As regards the rotational elasticity of this hydrodynamical æther, on which we have made all radiative and electrical phenomena depend, it was objected, in 1862, by Sir George Stokes* to MacCullagh's æther, that a medium of that kind would leave unbalanced the tangential surface-tractions on an element of volume, and therefore could not be in internal equilibrium; and this objection has usually been recognised, and has practically led to MacCullagh's theory of light being put aside, at any rate in this country. Now, it has been already mentioned that a precisely equivalent objection will apply to the elasticity actually produced by a gyrostatic distribution of momentum in an ordinary solid medium, the only difference in the circumstances being that in the latter case the rotational elasticity is proportional to the angular velocity and not to the angular displacement;† and this remark suggests that there must be some way out of the difficulty. If we consider the laws of motion, stated in Newton's manner with reference to absolute space and absolute time, as fundamental principles, then it is also a fundamental principle that the energy of a spinning gyrostat has reference to absolute space, and is not relative to the material system which contains it. The gyrostat may be considered as a kind of connexion binding that system to absolute immovable space by means of the forcive which it opposes to rotation; and this is the reason why the element of mass in a gyrostatic medium remains in equilibrium with its translational kinetic reactions, although the tractions of the surrounding parts on its surface are unbalanced and result in a couple. If this mode of viewing the subject is regarded as incongruous, then we must discard from dynamics the notion of absolute space, and we must set out in quest of some transcendental explanation of the directional forcives in rotational systems. In any case the general Lagrangian dynamical procedure applies precisely to the gyrostatic medium we have here taken as an illustration: nor, probably, would its application to MacCullagh's æther be questioned, once the preliminary objection was removed.

* [I am informed by Sir George Stokes that in the above criticism he contemplated only media of which the elements are self-contained, and devoid of internal motions.]

† For a detailed discussion of equilibrium and wave-propagation in such a medium, see 'Proc. Lond. Math. Soc.,' 1890.

This question may also be instructively illustrated from another side, by the consideration of an actual medium which possesses precisely the rotational elasticity of MacCullagh's æther. I allude to a solid medium with small magnets interspersed through it in any arbitrary manner, but so that in any single element of volume there is some regularity in their orientation. If this medium when unstrained is in equilibrium in a magnetic field, then when an element of it is displaced rotationally it will be acted on by a bodily couple arising from this external field; and therefore the surface tractions on the element would, in the presence of this couple, be unbalanced. Here the disturbing cause is a magnetic forcive arising either from the medium as a whole or from some external system; it has to be considered as of a statical character, that is, the velocity of propagation of the magnetic action is supposed to be indefinitely great compared with the velocity of propagation of any disturbances that are under discussion; the magnetic influence of the whole system is supposed to be instantaneously brought to bear on the element, and not merely the influence of the surrounding parts. On this saving hypothesis, the magnetic energy is here also correctly localised, for dynamical purposes, in the element of volume of the medium, and the Lagrangian method has perfect application to the mathematical analysis of its phenomena.

Now, in the case of the æther we have at hand a *vera causa* precisely of this kind. The cause of the phenomena of gravitation has hitherto remained perfectly inscrutable. Though the present order of ideas forbids us to consider it otherwise than as propagated in time, yet all we know of its velocity of propagation is the demonstration by Laplace that it must, at the very least, exceed the velocity of propagation of light in the same kind of proportion as the latter velocity exceeds that of ordinary motions of matter. It is not unphilosophical to assume that an explanation of gravitation might carry along with it the explanation of the fact that the tangential tractions on an element of the strained æther are unbalanced. The dynamical phenomena of mass in matter would appear to be analytically explicable by the addition of a rotational part to the kinetic energy of the element of the medium; such a term is of course practically null except in the vortex rings.

In all that has been hitherto said we have kept clear of the complication of viscous forces; but in order to extend our account to the phenomena of opacity in the theory of radiation and of electric currents in ordinary conductors, it is necessary to introduce such forces and make what we can of them on general principles. It is shown that the introduction of the dissipation function into dynamics by Lord Rayleigh enables us to amend the statement of the fundamental dynamical principle, the law of Least Action, so as to include

in it the very extensive class of viscous forces which are proportional to absolute or relative velocities of parts of the system. This class is the more important because it is the only one that will allow a simple wave to be propagated through a medium with period independent of its amplitude; if the viscous forces that act in light propagation were not of this kind, then on passing a beam of homogeneous light through a metallic film it should emerge as a mixture of lights of different colours. The viscous forces being thus proved by the phenomena of radiation to be derived from a dissipation function, it is natural to extend the same conclusion to the elastic motions of slower periods than radiations, which constitute ordinary electric disturbances. We thus arrive, by way of an optical path, at Joule's law of dissipation of electric energy, and Ohm's linear law of electric conduction, and the whole theory of the electrodynamics of currents flowing in ordinary conductors; though the presumption is that the coefficients which apply to motions of long period are not the same as those which apply to very rapid oscillations, the characters of the matter-vibrations that are comparable in the two cases being quite different.* If it is assumed that the form of the dissipation function is the same for high frequencies as for low ones, we obtain the ordinary theory of metallic reflexion, which differs from the theory of reflexion at a transparent medium simply by taking the refractive index to be a complex quantity, as was done originally by Cauchy, and later for the most general case by MacCullagh. And, in fact, we could not make a more general supposition than this for the case of isotropic media; while for crystalline media the utmost generality would arise merely from assuming the principal axes of the dissipation function to be different from those of the rotational elasticity, a hypothesis which is not likely to be required.

It has been pointed out, originally by Lord Rayleigh, that to fit this theory to the facts of metallic reflexion it is necessary to take the real part of the index of refraction of the metals to be a negative quantity, which can hardly be allowed on other grounds, as it would imply instability of the medium. We might indeed, following the view of Willard Gibbs and others, imagine an interaction between the light wave and the free vibrations of the atomic electric charges, and through them the chemical vibrations of the atoms, owing to proximity of their periods; and we might possibly conceive the electric medium to be, so to speak, held together by this kind of support. But I think there is another and simpler alternative that

* It is interesting to notice that, already in his memoir of 1864, Maxwell is struck by the identity of the coefficients of the free æther for all periods, which "shows how perfect and regular the elastic properties of the medium must be when not encumbered with any matter denser than air."

merits examination; we might conceive the opacity at the surface to be so great that a sensible part of the light is lost before it has penetrated more than a very small fraction of a wave-length. In the extreme case of electric waves of finite length reflected from metals, the absorption is complete in a very small fraction of the wave-length, and the result is total reflexion, as from a vacuum; on the other hand, if the opacity is but slight, the phenomena ought to agree approximately with those of transparent media. It seems worth while to examine the consequences of assuming that the optical phenomena of metallic reflexion are nearer the first of these limiting cases than the second. It seems worth while also to compare the facts for some medium not so opaque as metals with the formulæ of Cauchy and MacCullagh; the examples of tourmaline crystal, and some of the aniline dyes which exhibit selective absorption, suggest themselves as affording crucial tests.*

The considerations which have here been explained amount to an attempt to extend the regions of contact between three ultimate theories which have all been already widely developed, but in such a way as not to have much connexion with one another. These theories are Maxwell's theory of electric phenomena, including Ampère's theory of magnetism and involving an electric theory of light, Lord Kelvin's vortex-atom theory of matter, and the purely dynamical theories of light and radiation that have been proposed by Green, MacCullagh, and other authors. It is hoped that a sufficient basis of connexion between them has been made out, to justify a re-statement of the whole theory of the kind here attempted, notwithstanding such errors or misconceptions on points of detail as will unavoidably be involved in it.

[While writing this summary it had escaped my memory that Lord Kelvin has proposed a gyrostatic adynamic medium which forms an exact representation of a rotationally elastic medium such as has been here described.† If the spinning bodies are imbedded in the æther so as to partake fully in its motion, the rotational forcive due to them is proportional jointly to the angular momentum of a gyrostat and the angular velocity of the element of the medium, in accordance with what is stated above. But if we consider the rotators to be free gyrostats of the Foucault type, mounted on gymbals of which the outer frame is carried by the medium, there will also come into play a steady rotatory forcive, proportional jointly to the square of the an-

* [An alternative view, in many respects preferable, is supplied by the assumption, with Sir George Stokes, of the existence in metals of an *adamantine* property, such as was discovered by Airy for the diamond. *Cf.* Sir G. G. Stokes, 'Proc. Roy. Soc.,' February, 1883.]

† Lord Kelvin (Sir W. Thomson), 'Comptes Rendus,' Sept. 16, 1889; 'Collected Papers,' vol. 3, 1890, p. 467.

gular momentum of the gyrostat and to the absolute angular displacement of the medium. An ideal gyrostatic cell has been imagined by Lord Kelvin in which the coexistence of pairs of gyrostats spinning on parallel axles in opposite directions cancels the first of these forcives, thus leaving only a static forcive of a purely elastic rotational type. The conception of an æther which is sketched by him on this basis* is essentially the same as the one we have here employed, with the exception that the elemental angular velocity of the medium is taken to represent magnetic force, and in consequence the medium fails to give an account of electric force and its static and kinetic manifestations. A gyrostatic cell of this kind has internal freedom, and therefore free vibration periods of its own; it is necessary to imagine that these periods are very small compared with the periods of the light waves transmitted through the medium, in order to avoid partial absorption. The propagation of waves in this æther, having periods of the same order as the periods of these free vibrations, would of course be a phenomenon of an altogether different kind, involving diffusion through the medium of energy of disturbed motion of the gyrostats within the cells.

Lord Kelvin has shown that a fluid medium, in turbulent motion owing to vorticity distributed throughout it, would also possess rotational elasticity provided we could be assured of its permanence. Professor G. F. Fitzgerald proposes to realise such a medium by means of a distribution of continuous vortex filaments, interlacing in all directions; if the vorticities of the filaments in an element of volume are directed indifferently in all directions, the motional part of the kinetic forcive on the element, which depends on the first power of the vorticity, will be null, while the positional part depending on the square of the vorticity will remain, just as in the gyrostatic medium above considered. The atoms may now be imagined to consist of vortex rings making their way among these vortex filaments, and thus a very graphic and suggestive scheme is obtained; the question of stability is however here all-important. No ultimate theory can be final; and schemes of the kind discussed in this paper may not inaptly be compared to structural formulæ in modern chemistry; they bind together phenomena that would otherwise have to be taken as disconnected, though they are themselves provisional and may in time be replaced by more perfect representations.

The electric interpretation of MacCullagh's optical equations, which forms the basis of this paper, was first stated so far as I know by Professor G. F. FitzGerald, 'Phil. Trans.,' 1880. I have recently learned, from a reference in Mr. Glazebrook's Address, British Association, 1893, that an electric development of Lord Kelvin's

* Lord Kelvin (Sir W. Thomson), 'Collected Papers,' vol. 3, 1890, pp. 436—472.

rotational æther has been essayed by Mr. Heaviside, who found it to be unworkable as regards conduction-current, and not sufficiently comprehensive ('Phil. Trans.,' 1892, § 16; 'Electrical Papers,' vol. 2, p. 543). A method of representing the phenomena of the electric field by the motion of tubes of electric displacement has been developed by Professor J. J. Thomson, who draws attention to their strong analogies to tubes of vortex motion ("Recent Researches . . .," 1893, p. 52).

Professor Oliver Lodge has kindly looked for an effect of a magnetic field on the velocity of light, but has not been able to detect any, though the means he employed were extremely searching; the inference would follow, on this theory, that the motion in a magnetic field is very slow, and the density of the medium correspondingly great.—*December* 18, 1893.]

IV. "On Copper Electrolysis *in Vacuo*." By WILLIAM GANNON, M.A. Communicated by Professor SCHUSTER, F.R.S. Received November 14, 1893.

[Publication deferred.]

V. "Note on the Action of Copper Sulphate and Sulphuric Acid on Metallic Copper." By ARTHUR SCHUSTER, F.R.S. Received November 14, 1893.

[Publication deferred.]

VI. "On a Chart of the Symmetrical Curves of the Three-Bar Motion." By W. BRENNAND. Communicated by C. B. CLARKE, F.R.S. Received November 17, 1893.

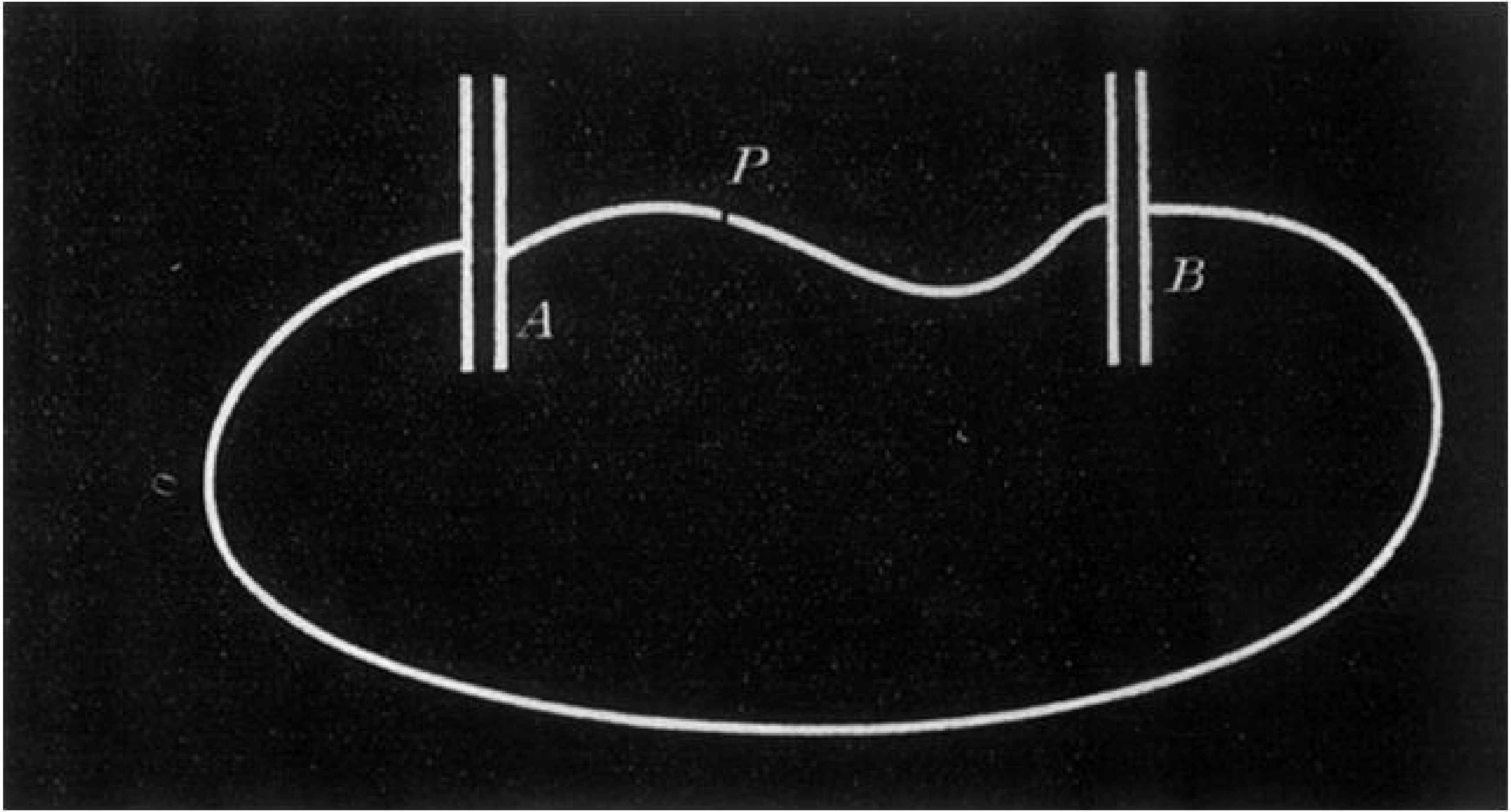
Presents, *December* 7, 1893.

Transactions.

Baltimore:—Medical and Chirurgical Faculty of the State of Maryland. Transactions. 94th Annual Session. 8vo. *Baltimore* 1892. The Faculty.

Batavia:—Koninkl. Natuurkundige Vereeniging. Natuurkundig Tijdschrift. Deel LII. 8vo. *Batavia* 1893. The Society.

Berlin:—Gesellschaft für Erdkunde. Zeitschrift. Bd. XXVIII. No. 3. 8vo. *Berlin* 1893. The Society.



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